

Problem 3

Nice Integral

Note that

$$\begin{aligned}\mathbb{E}[(X-1)^n] &= \int_0^\infty \lambda e^{-\lambda x} (x-1)^n dx \\ &= [-e^{-\lambda x} (x-1)^n]_{x=0}^{x=\infty} + \int_0^\infty e^{-\lambda x} n (x-1)^{n-1} dx \\ &= (-1)^n + \frac{n}{\lambda} \int_0^\infty \lambda e^{-\lambda x} (x-1)^{n-1} dx.\end{aligned}$$

If we let

$$a_n = \int_0^\infty \lambda e^{-\lambda x} (x-1)^n dx$$

we get the recursive relation

$$\begin{aligned}a_n &= (-1)^n + \frac{n}{\lambda} a_{n-1}, \text{ with} \\ a_0 &= \int_0^\infty \lambda e^{-\lambda x} dx = 1.\end{aligned}$$

You can write a program that solves this recursive relation for you.